3DV 2020

8th International Conference on

3D Vision

http://3dv2020.dgcv.nii.ac.jp/

3DV 2020

Fast Simultaneous Gravitational Alignment of Multiple Point Sets

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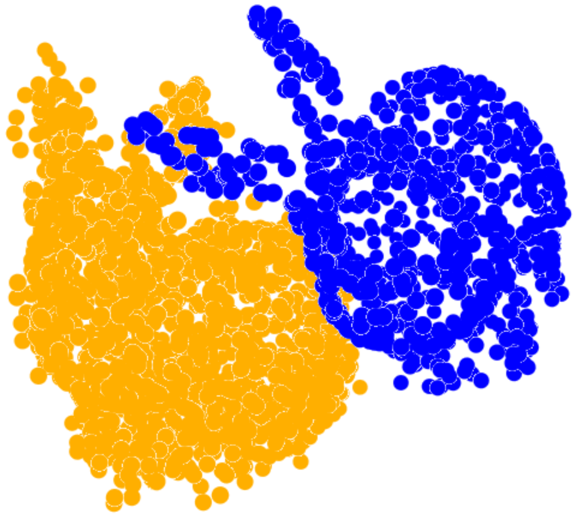
max planck institut informatik

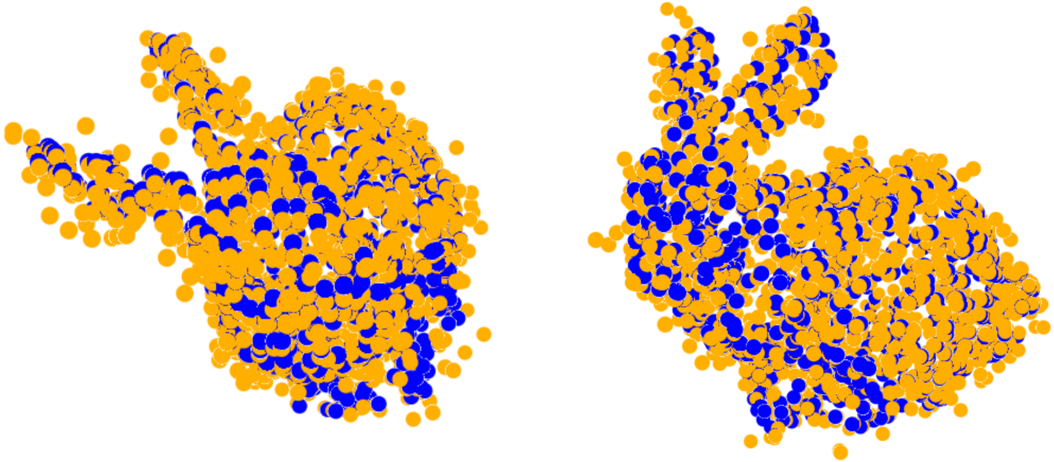
Graphics Vision & Video

Saarland Informatics Campus SIC

Rigid Point Set Alignment



input point sets

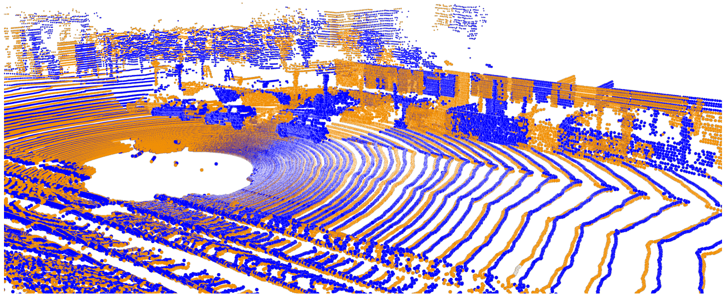


registration result

Rigid Point Set Alignment

[1]

[2]



[1]

[3]

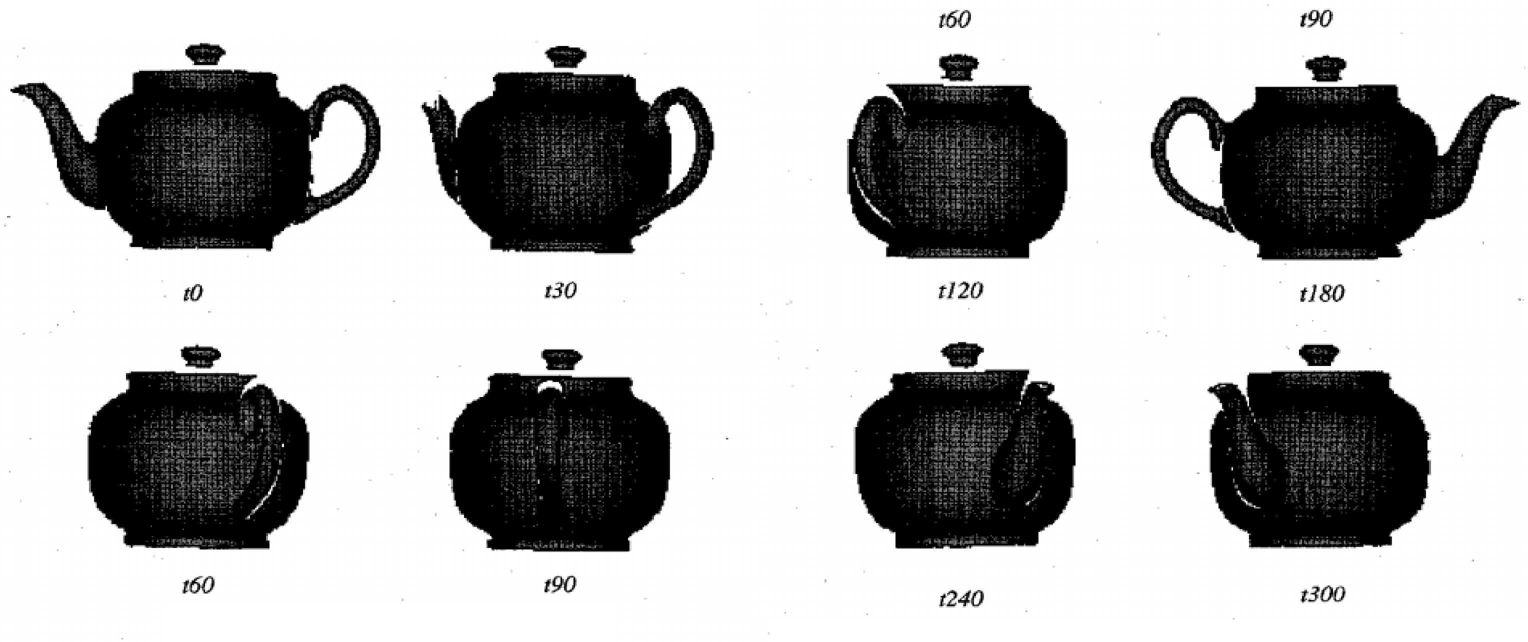
structured light

RGB-D

LIDAR

[1] V. Golyanik *et al.* Accelerated Gravitational Point Set Alignment with Altered Physical Laws. In *ICCV*, 2019.
[2] G. D. Evangelidis and R. Horaud. Joint alignment of point sets with batch and incremental expectation-maximization. TPAMI, 2018.
[3] F. Järemo Lawin *et al.* Density adaptive point set registration. In *CVPR*, 2018.

Rigid Point Set Alignment



Bergevin *et al.* Towards a general multi-view registration technique. TPAMI, 1996.

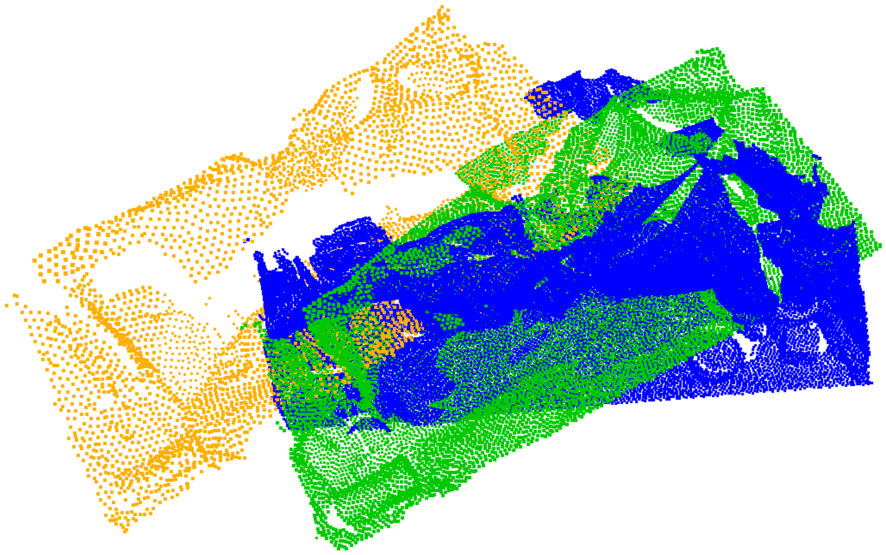
Rigid Point Set Alignment

ten point sets

registration result

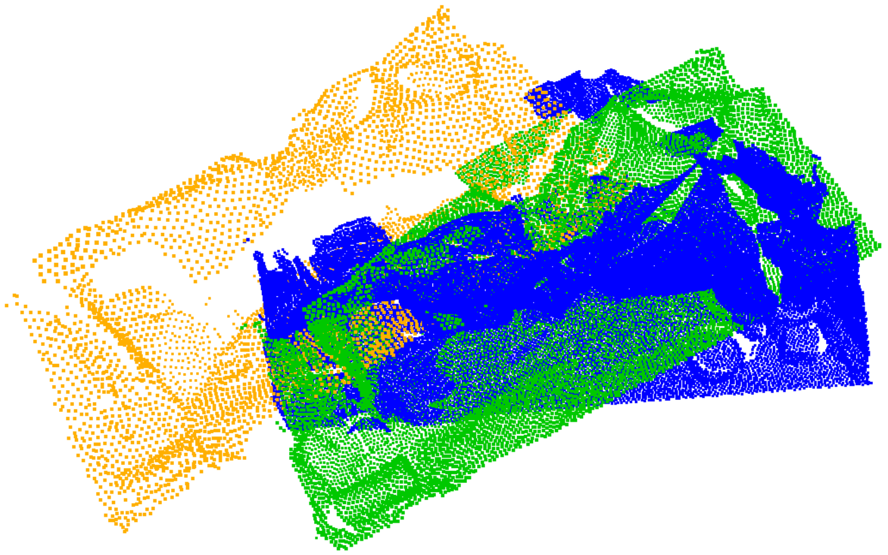
G. D. Evangelidis and R. Horaud. Joint alignment of point sets with batch and incremental expectation-maximization. TPAMI, 2018.

Multi-Body Gravitational Approach

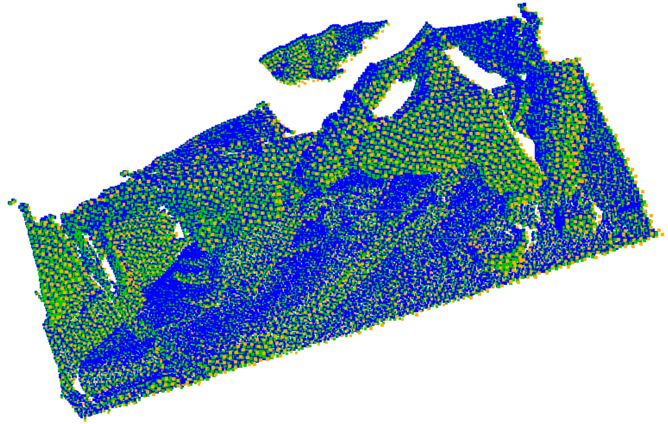


three point sets

Multi-Body Gravitational Approach

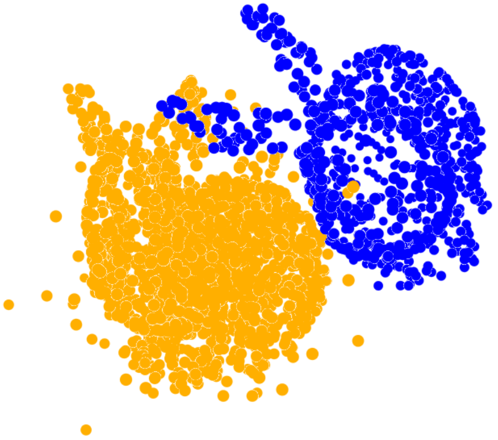


three point sets

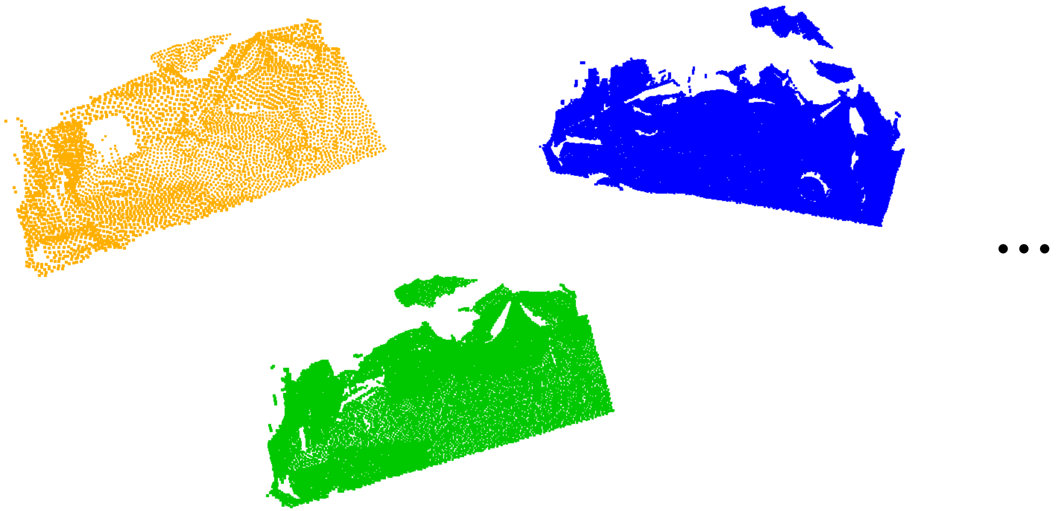


registration result

Rigid Point Set Alignment



fixed **reference** and
a transformed template



no reference, all point sets are handled on par

Related Work

Descriptor-Based Approaches

FPFH: Rusu *et al.*, ICRA, 2009.
FGR: Zhou *et al.*, ECCV, 2016.
DGR: Choy *et al.*, CVPR, 2020.
Gojcic and Zhou *et al.*, CVPR 2020.

Iterative Closest Points

ICP: Besl and McKay, TPAMI, 1992,
Chen and Medioni, IVC, 1992
Bergevin *et al.*, TPAMI, 1996.
Eggert *et al.*, CVIU, 1996.
SoftAssign: Gold *et al.*, Pat. Rec., 1997.
LM-ICP, Fitzgibbon, BMVC, 2001
MA-ICP, Govindu and Pooja, TIP, 2014.

Probabilistic Methods

MPM: Chui and Rangarajan, MMBIA, 2000.
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CPD: Myronenko and Song, TPAMI, 2010.
GMM-Reg: Jian and Vemuri, TPAMI, 2011.
JRMPC: Evangelidis and Horaud, TPAMI, 2018.

Physics-Based Methods

SDTM: Deng *et al.*, CVPR, 2014.
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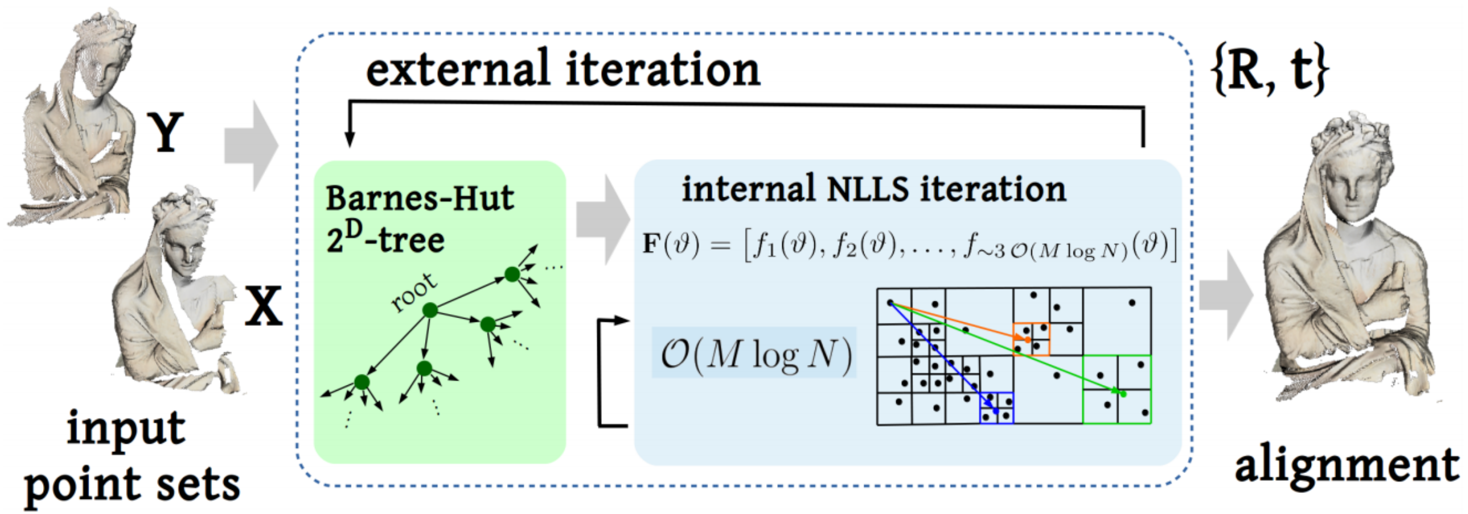
Contributions

i/ The first **gravitational method for multi-body point set alignment**
ii/ **Acceleration of globally multiply-linked point interactions** with a 2^D-tree; this data structure enables **a new fast shape signature** based on polynomial fitting
iii/ Experimental **evaluation with SotA results**

BHRGA Algorithm



$$\mathbf{E}(\mathbf{R}, \mathbf{t}) = \sum_i \sum_j m_{\mathbf{y}_i} m_{\mathbf{x}_j} \|\mathbf{R}\,\mathbf{y}_i + \mathbf{t} - \mathbf{x}_j\|_2$$

V. Golyanik *et al.* Accelerated Gravitational Point Set Alignment with Altered Physical Laws. In *ICCV*, 2019.

Multi-Body Gravitational Approach

Generalisation of gravitational alignment for L point clouds:

$$\mathbf{E}(\mathbf{T}) = \sum_{l=1}^{L} \sum_{i=1}^{|\mathbf{Y}_l|} \sum_{\substack{\mathbf{p}_j \in \\ \{\mathbf{Y} \setminus \mathbf{Y}_l\}}} \left(m_{\mathbf{p}_i}^{l} \, m_{\mathbf{p}_j} \left\| g(\mathbf{T}_l, \mathbf{p}_i^l) - \mathbf{p}_j \right\|_2 \right)$$

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For each point set $\boldsymbol{Y}_l$

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For each point set $\mathbf{Y}_l$, evaluate the gravitational potential at point $\mathbf{p}_i^l \in \mathbf{Y}_l$

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For each point set $\boldsymbol{Y}_l$, evaluate the gravitational potential at point $\mathbf{p}_i^l \in \boldsymbol{Y}_l$ induced by all other points $\mathbf{p}_j$ from all other point sets $\{\boldsymbol{Y} \setminus \boldsymbol{Y}_l\}$.

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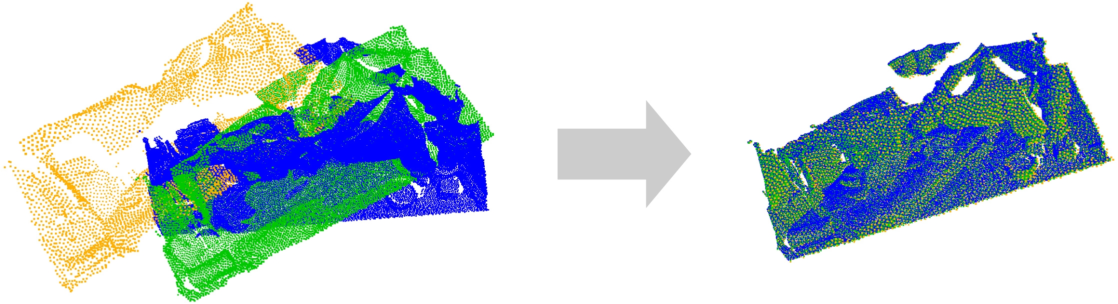
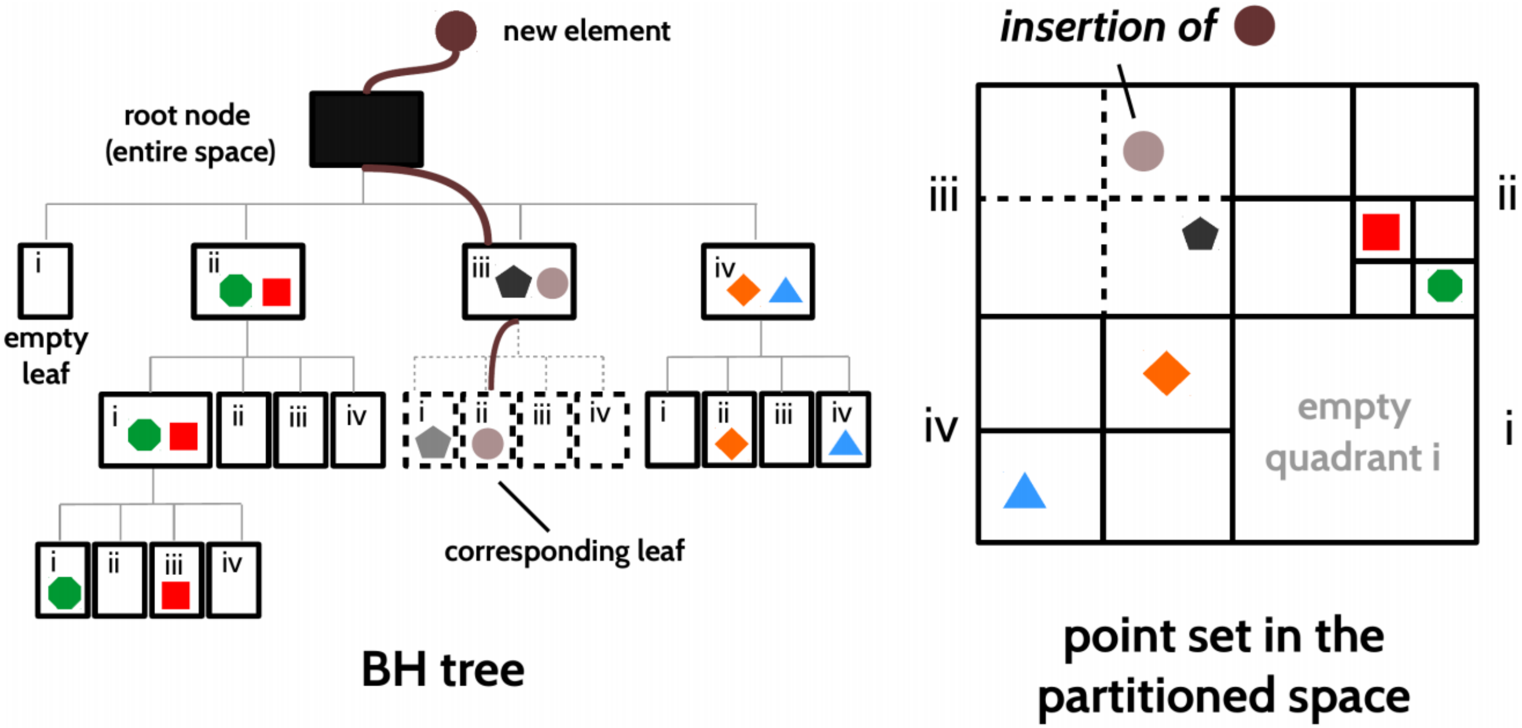


Acceleration of Point Interactions

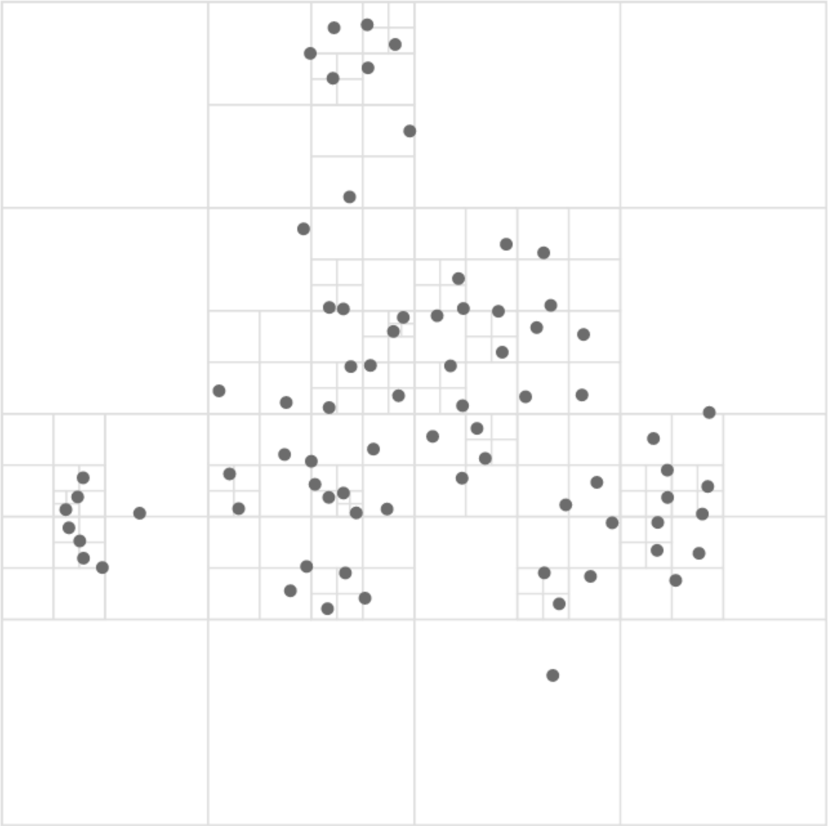


J. Barnes and P. Hut. A hierarchical O(n-log-n) force calculation algorithm. Nature, 324, 1986.
V. Golyanik *et al.* Accelerated Gravitational Point Set Alignment with Altered Physical Laws. In *ICCV*, 2019.

Acceleration of Point Interactions

Visualiser: https://jheer.github.io/barnes-hut/

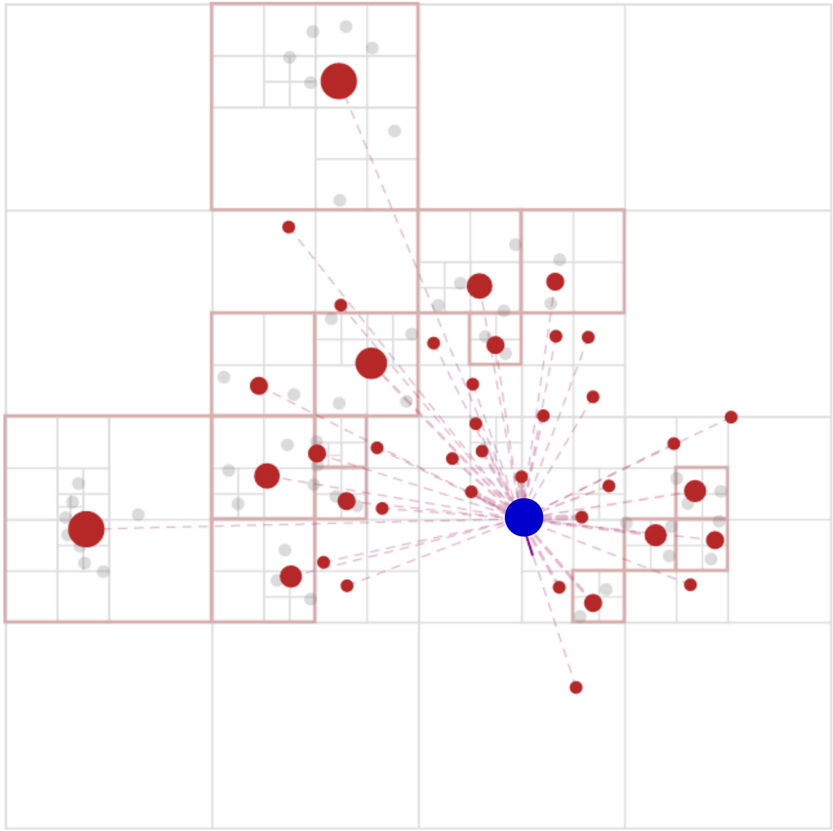
examplary 2D point set

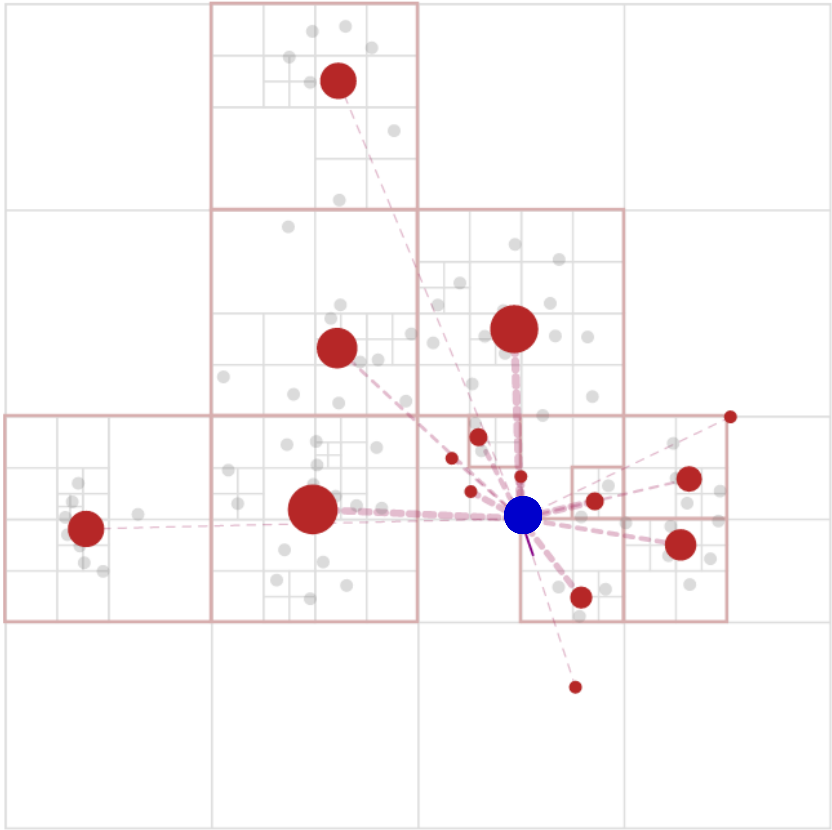


Barnes-Hut quadtree

Acceleration of Point Interactions

Visualiser: https://jheer.github.io/barnes-hut/





calculation of the force acting on the blue particle

Acceleration of Point Interactions

Gravitational potential energy with Barnes-Hut acceleration:

$$\mathbf{E}_{\mathbf{C}}(\mathbf{T}) = \sum_{l=1}^{L} \sum_{i=1}^{|\mathbf{Y}_l|} \sum_{\mathbf{c}_j \in \mathbf{C}_{l,i}} \left(m_{\mathbf{p}_i}^{l} \, m_{\mathbf{c}_j} \left\| g(\mathbf{T}_l, \mathbf{p}_i^l) - \mathbf{c}_j \right\|_2 \right)$$

Acceleration of Point Interactions

Gravitational potential energy with Barnes-Hut acceleration:

$$\mathbf{E}_{\mathbf{C}}(\mathbf{T}) = \sum_{l=1}^{L} \sum_{i=1}^{|\boldsymbol{Y}_l|} \boxed{\sum_{\mathbf{c}_j \in \mathbf{C}_{l,i}}} \left(m^l_{\mathbf{p}_i} \boxed{m_{\mathbf{c}_j}} \left\| g(\mathbf{T}_l, \mathbf{p}^l_i) - \boxed{\mathbf{c}_j} \right\|_2 \right)$$

For each point set $\boldsymbol{Y}_l$, evaluate the gravitational potential at point $\mathbf{p}^l_i \in \boldsymbol{Y}_l$ induced by the fetched clusters $\mathbf{c}_j \in \mathbf{C}_{l,i}$ from the 2^D-tree.

Complexity (Runtime and Memory)

Proposition 1. *The computational complexity of MBGA is* $\mathcal{O}(L\bar{N}\log(L\bar{N}))$, *where* L *is the total number of point sets, and* $\bar{N}$ *is the average number of points in each point set.*

Proposition 2. *The memory complexity of MBGA handling* $L\bar{N}$ *points is* $\mathcal{O}(L\bar{N}\log(L\bar{N}))$, *with the factor* $\log(L\bar{N})$ *attributable to the nodes in the* 2^D*-tree.*

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quasi-linear

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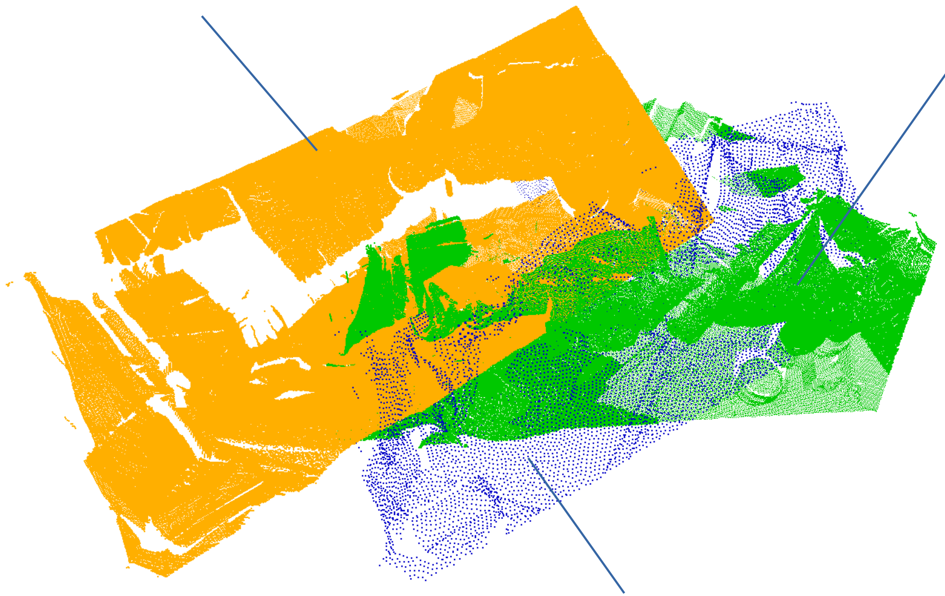
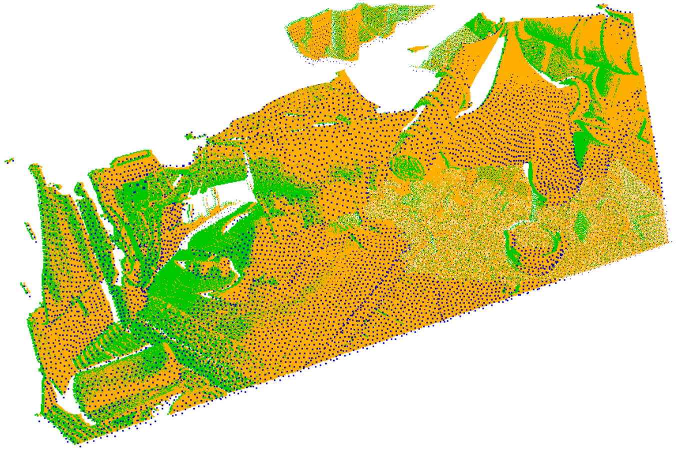
Multi-Body Gravitational Approach

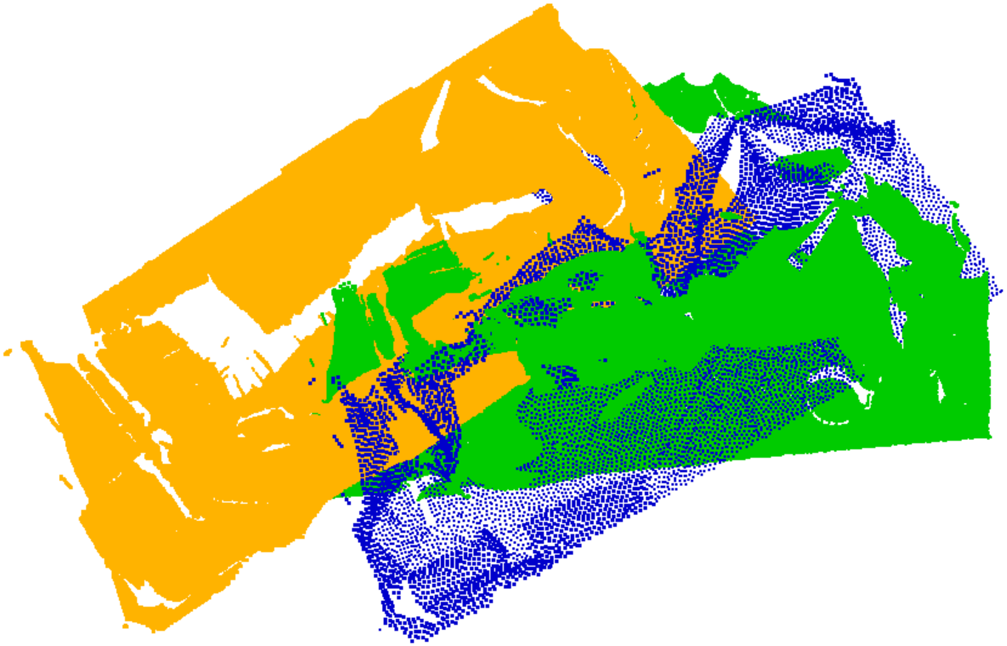


initialisation



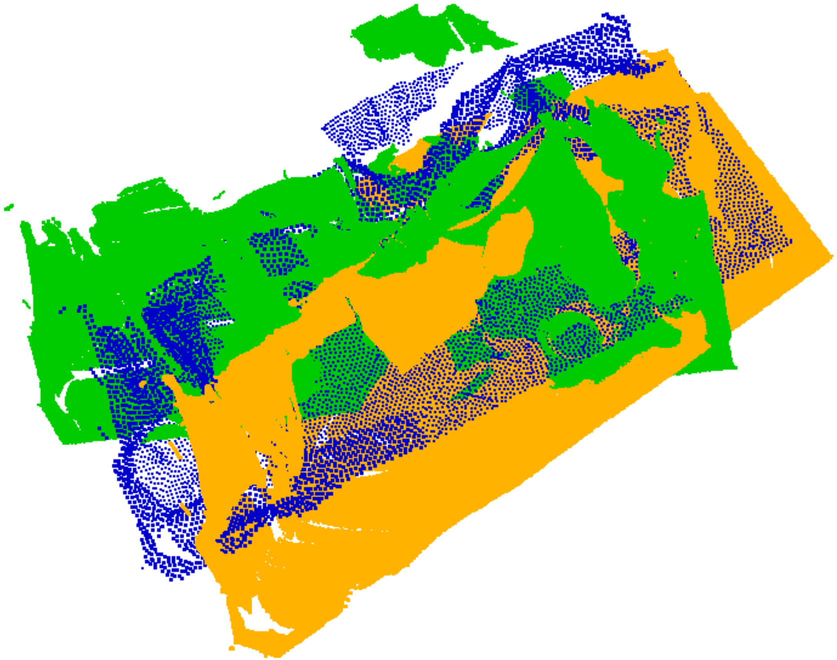
registration

Iterative NLLS Optimisation



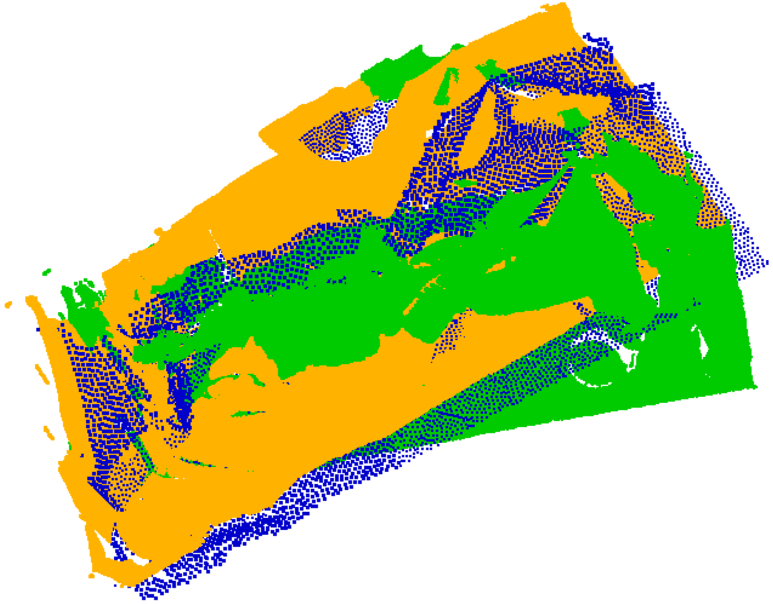
iteration 1

Iterative NLLS Optimisation



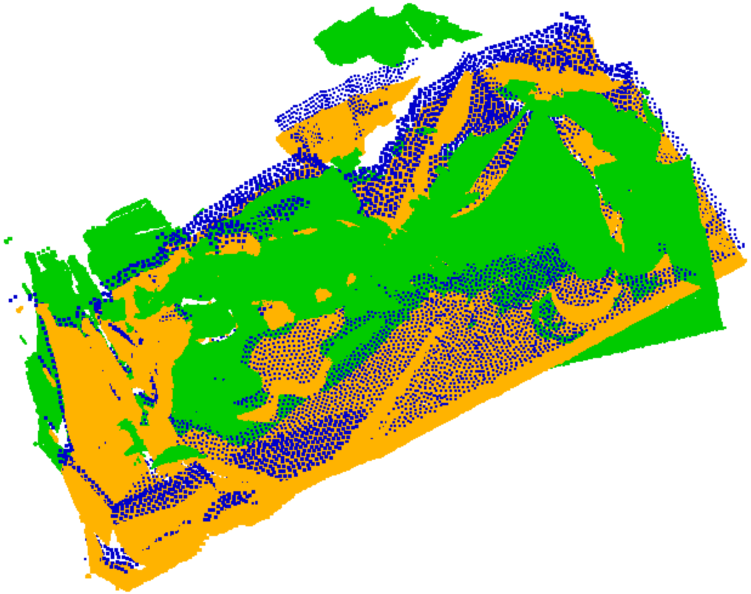
iteration 2

Iterative NLLS Optimisation



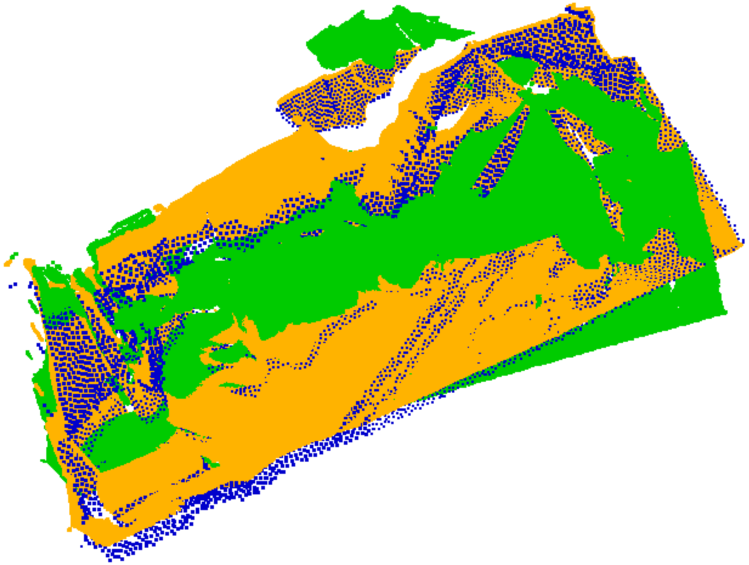
iteration 3

Iterative NLLS Optimisation



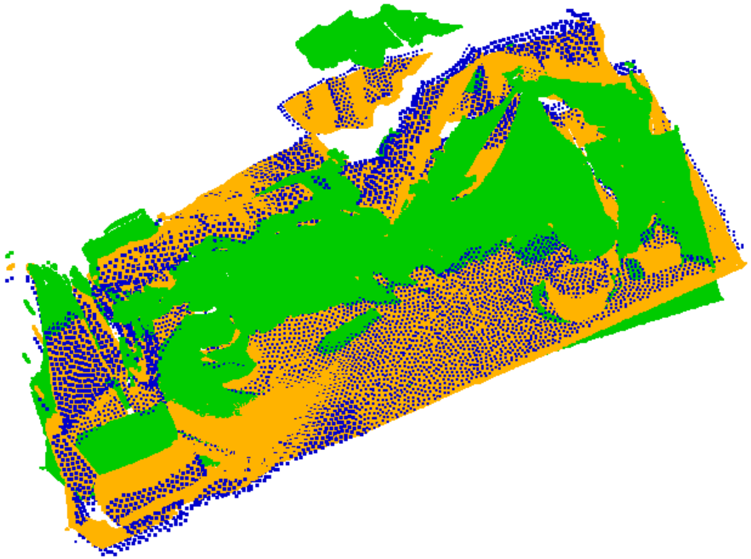
iteration 4

Iterative NLLS Optimisation



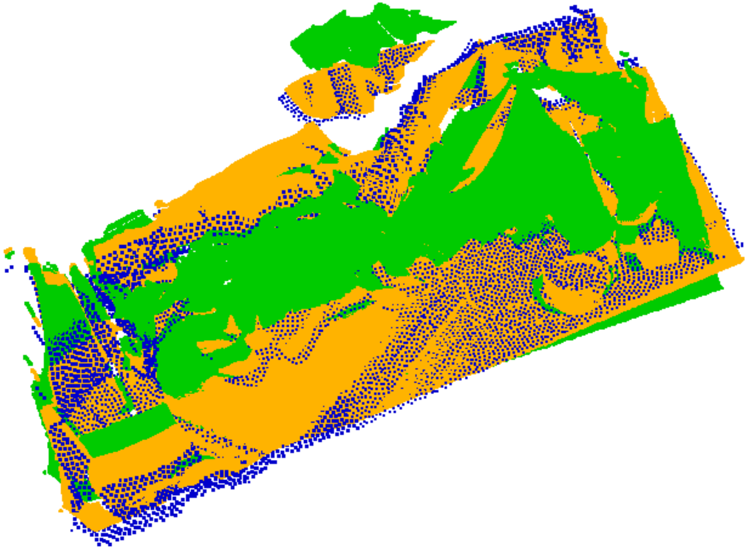
iteration 5

Iterative NLLS Optimisation



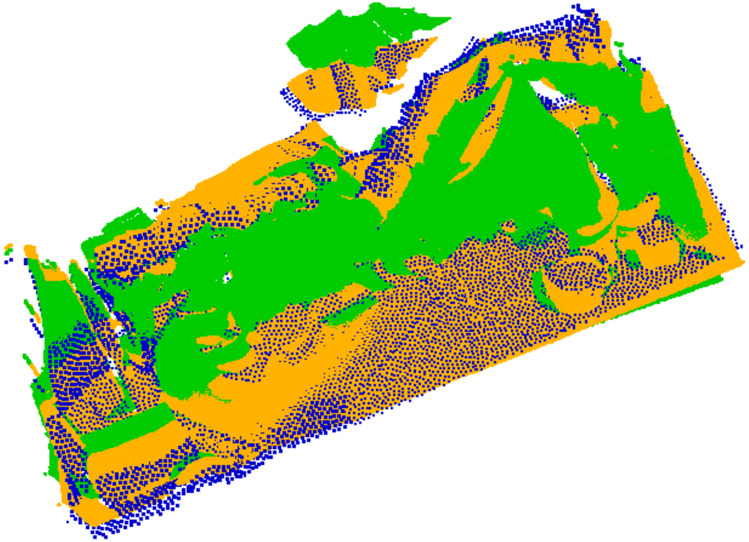
iteration 6

Iterative NLLS Optimisation



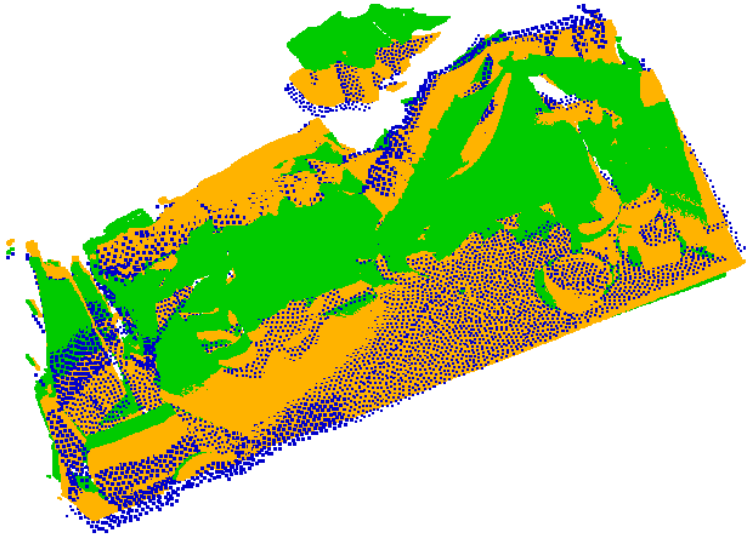
iteration 7

Iterative NLLS Optimisation



iteration 8

Iterative NLLS Optimisation



iteration 9

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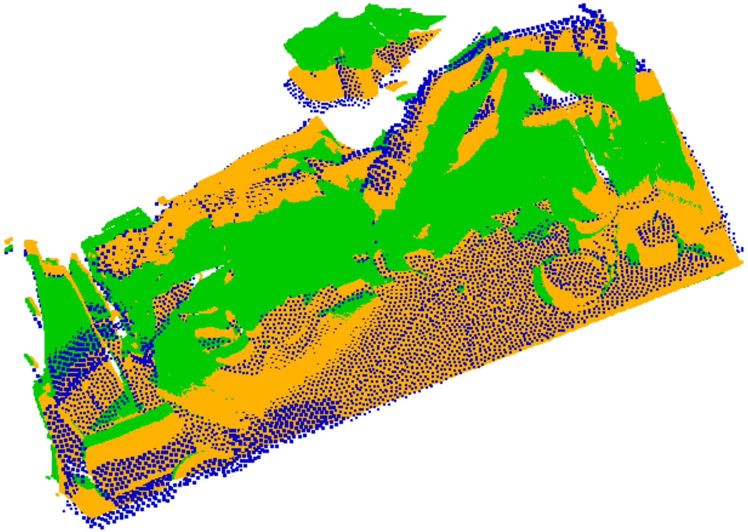

iteration 10

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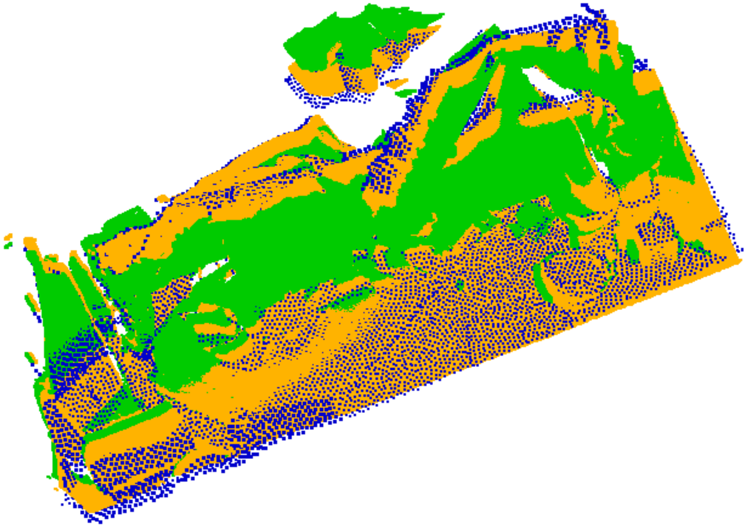

iteration 11

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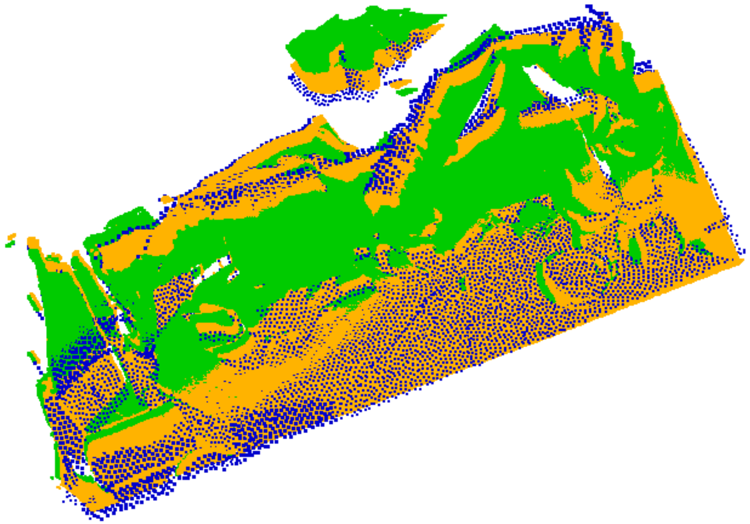

iteration 12

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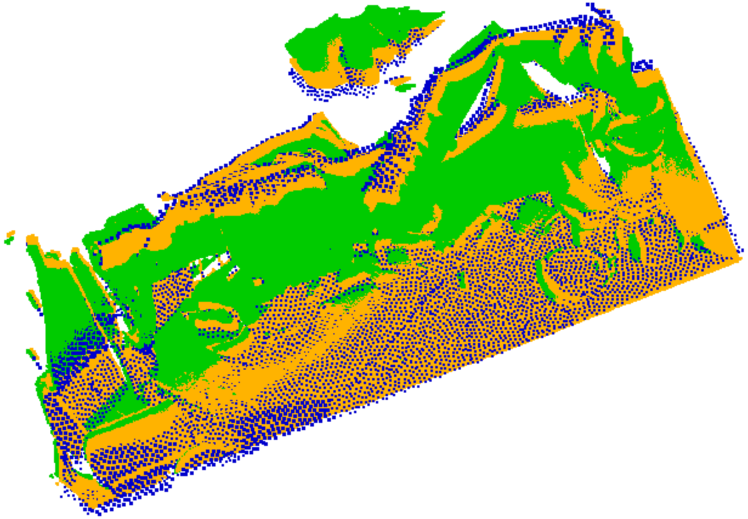

iteration 13

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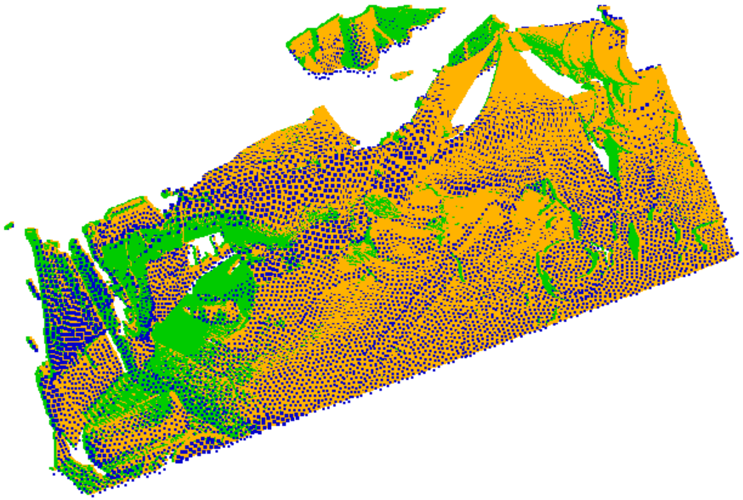


iteration 27

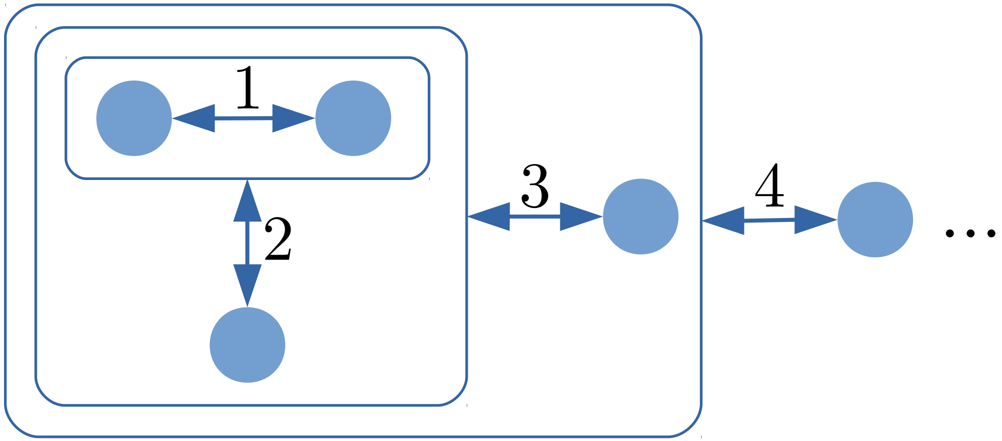
(converged)

Experimental Evaluation

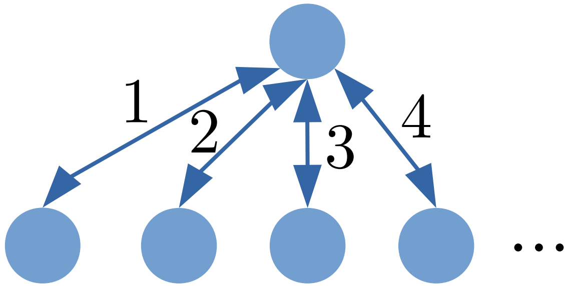
Two policies for the evaluation of pairwise methods:

Experimental Evaluation

Two policies for the evaluation of pairwise methods:



growing reference (GRef)



one-to-many (1-M)

Experimental Evaluation

The average 3D RMSE error:

$$e_{3D} = \binom{L}{2}^{-1} \sum_{\{i,j\}\in\Phi} \frac{\|g(\mathbf{T}_i, \boldsymbol{Y}_i) - g(\mathbf{T}_j, \boldsymbol{Y}_j)\|_{\mathcal{F}}}{\|g(\mathbf{T}_i, \boldsymbol{Y}_i)\|_{\mathcal{F}}}$$

Φ denotes all combinations of two point sets out of L

$\binom{L}{2} = |\Phi|$ is the total number of combinations

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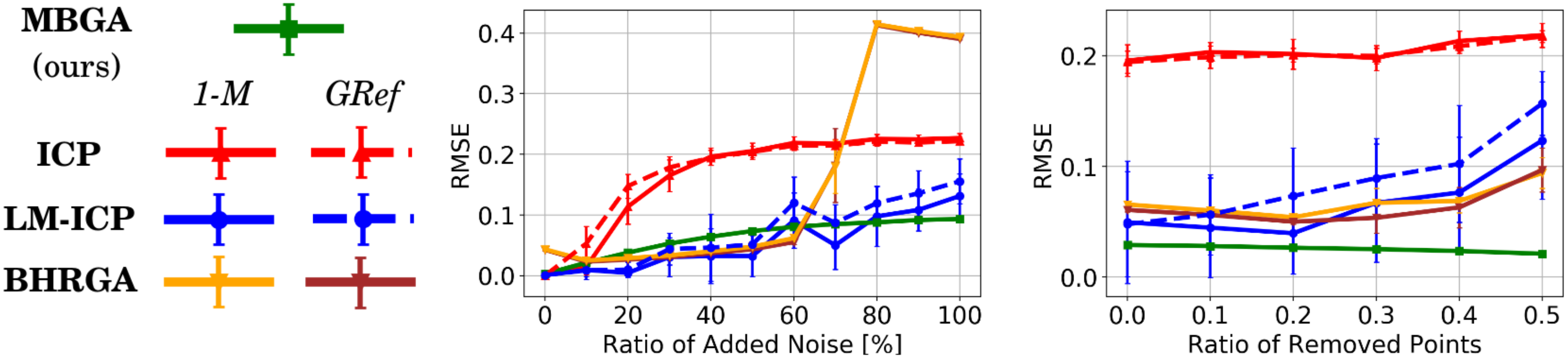
$+$ std. dev. of RMSE denoted by σ

Experimental Evaluation



comparison against pairwise algorithms

P. J. Besl and N. D. McKay. A method for registration of 3-d shapes. TPAMI, 1992.
A. Fitzgibbon. Robust registration of 2d and 3d point sets. In *BMVC*, 2003.
V. Golyanik *et al.* Accelerated Gravitational Point Set Alignment with Altered Physical Laws. In *ICCV*, 2019.

Experimental Evaluation

		ICP	LM-ICP	BHRGA	JRMPC	MBGA
N	e_{3D}	0.2244	0.1435	0.392	**1.5*E*-4**	**9.4*E*-2**
	σ	6.4*E*-3	3.7*E*-2	1.4*E*-3	**5.6*E*-5**	**1.1*E*-3**
R	e_{3D}	0.2181	0.1403	**9.6*E*-2**	**1.3*E*-3**	**2.1*E*-2**
	σ	8*E*-3	4.1*E*-2	**1.7*E*-2**	**3.6*E*-4**	**4.8*E*-4**

N – 100% of added uniform noise

R – 50% of randomly removed points

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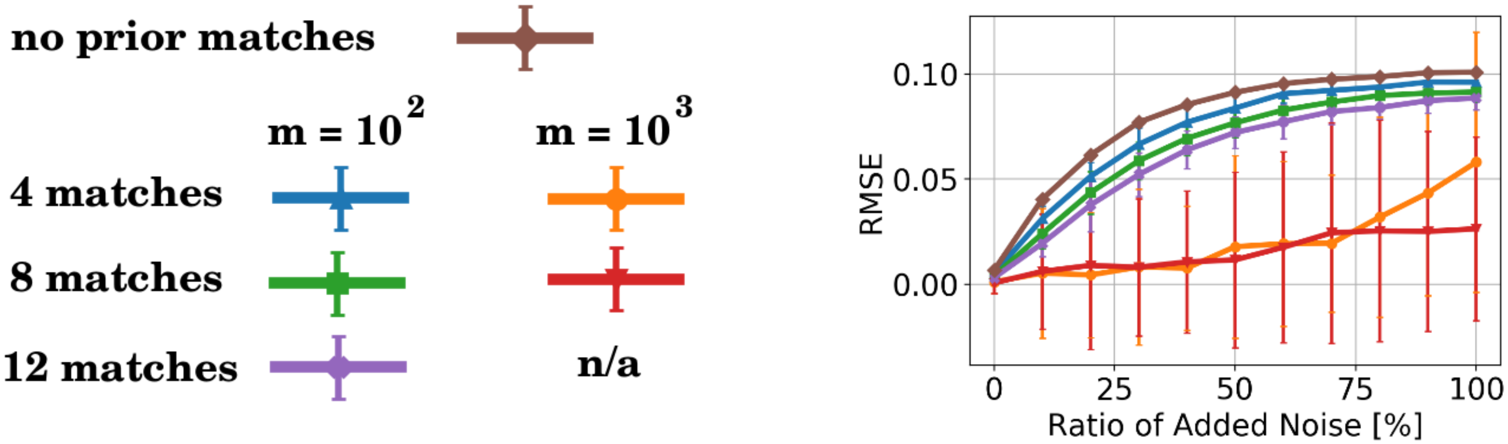
V. Golyanik *et al.* Accelerated Gravitational Point Set Alignment with Altered Physical Laws. In *ICCV*, 2019.

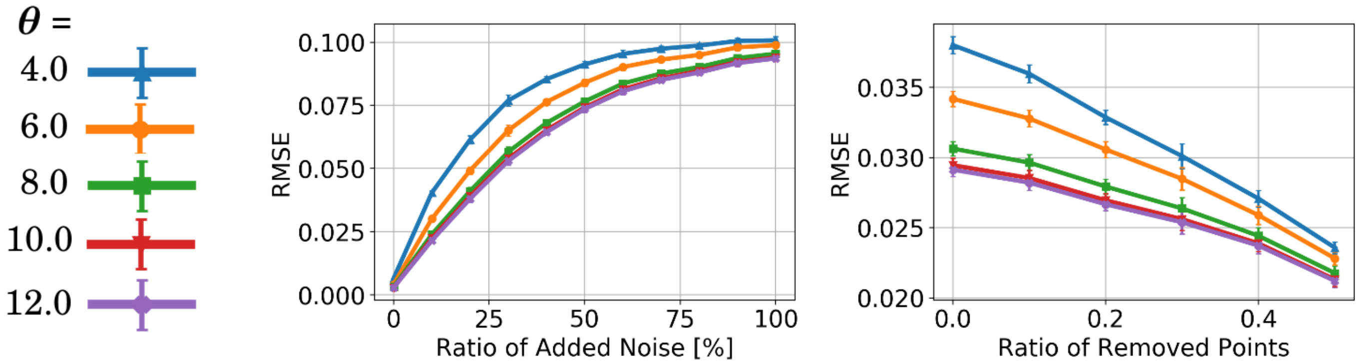
Experimental Evaluation



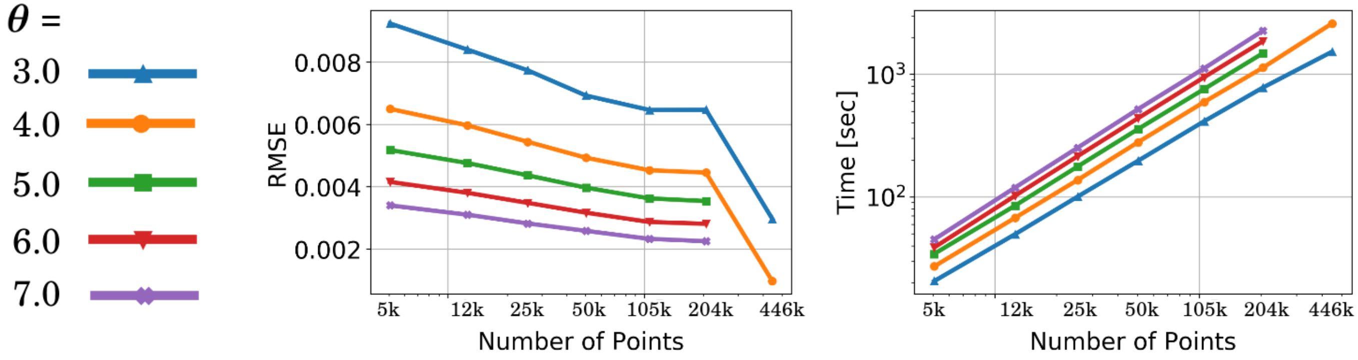
influence of prior correspondences

Experimental Evaluation

varying 2^D-tree threshold

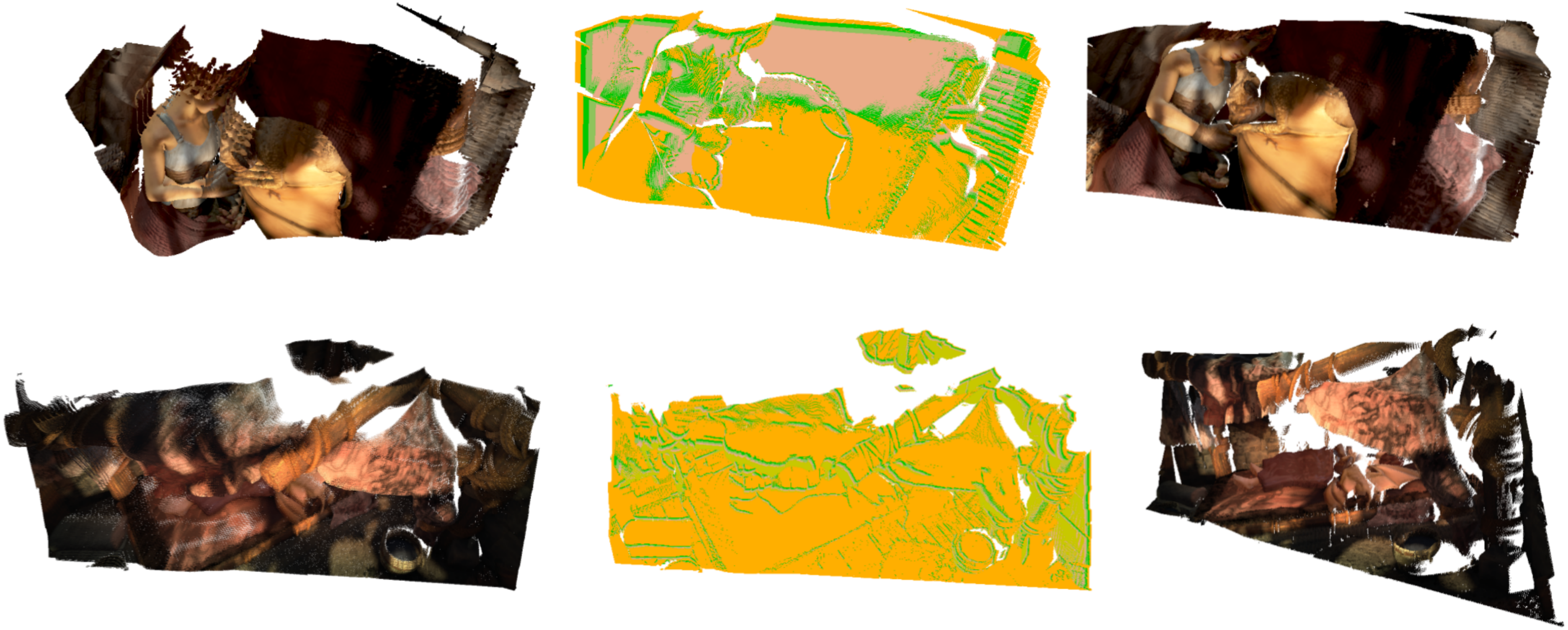


runtime *vs* accuracy



Experimental Evaluation (SINTEL)

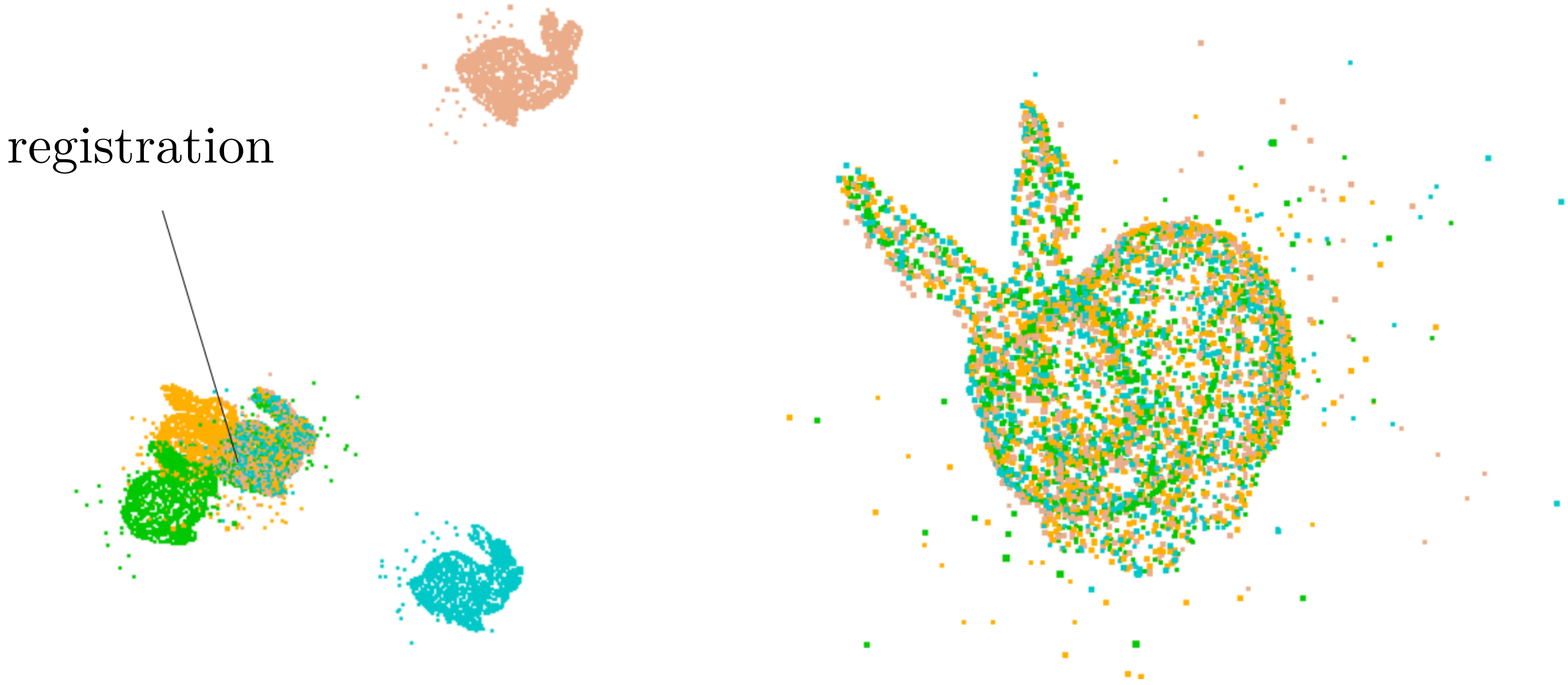
Dataset: D. J. Butler *et al.*, ECCV, 2012.



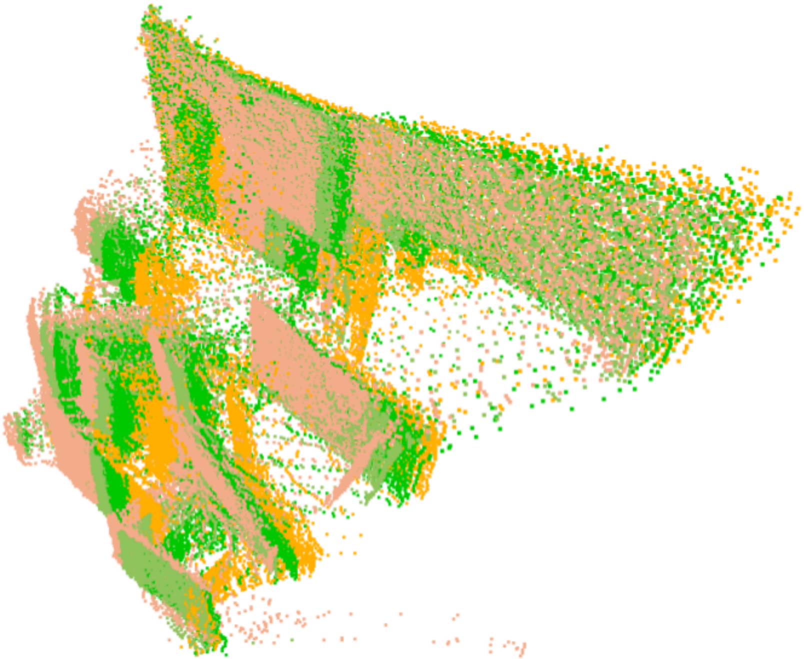
RGB-D stack

registrations

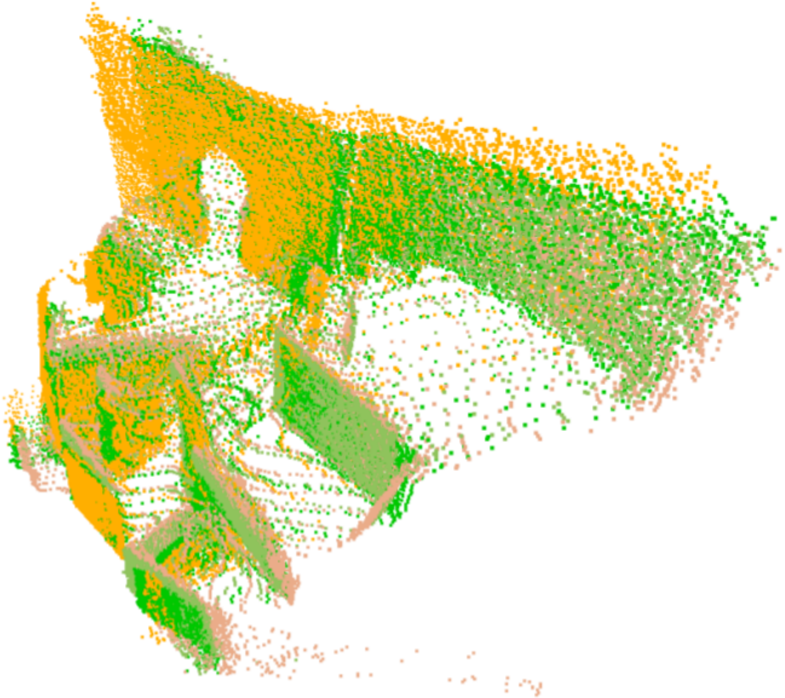
Experimental Evaluation



Experimental Evaluation (Depth Maps)

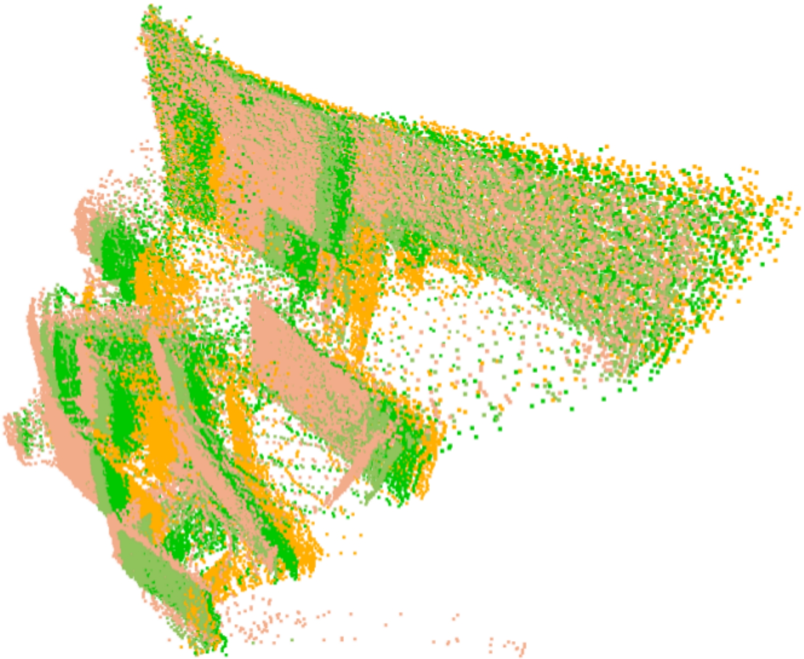


initialisation

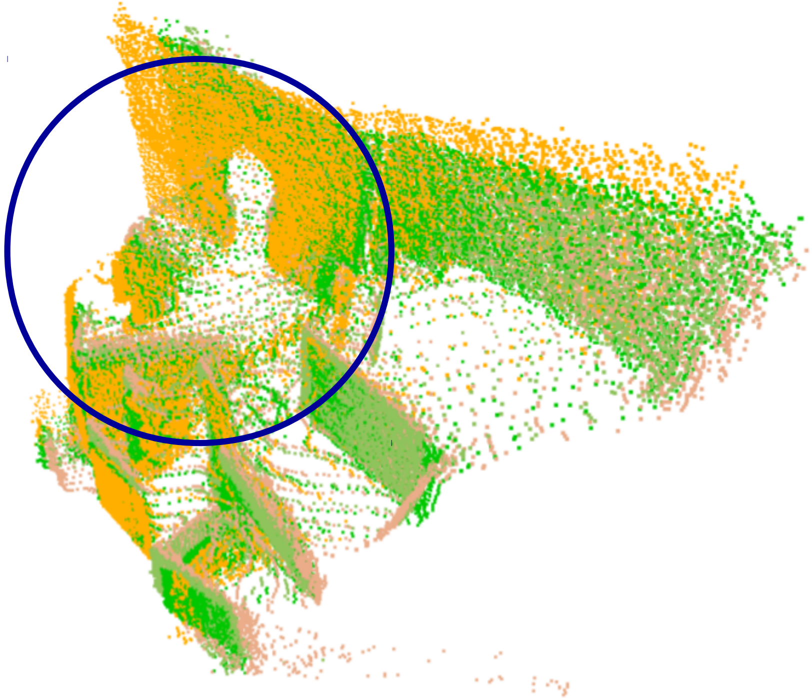


registration

Experimental Evaluation (Depth Maps)

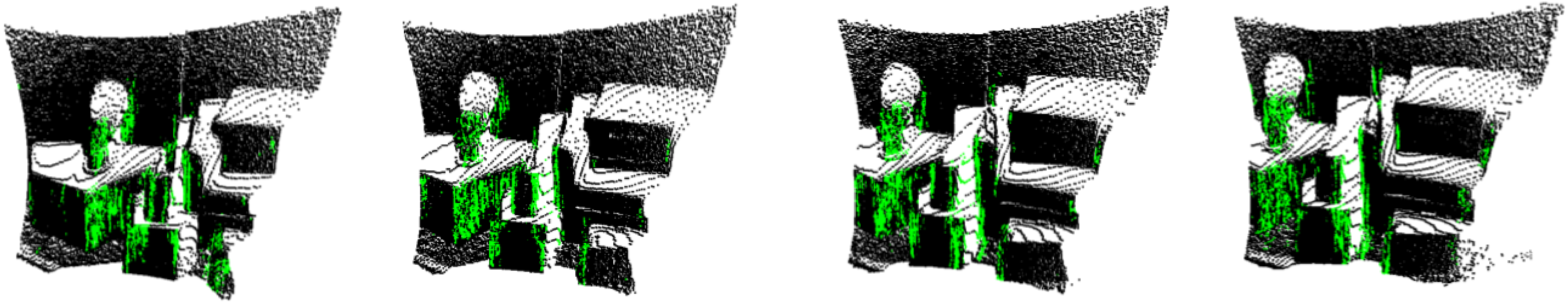


initialisation



registration

Experimental Evaluation (Depth Maps)



extracted shape signature induced by the 2^D-tree (in green)

Conclusions

+ The first gravitational method for multi-body point set alignment
+ All point sets are handled on par
+ Uses 2^D-tree for the acceleration of point interactions
+ Can align large point sets in a globally multiply-linked manner
+ Robust to large noise ratios and varying point sampling densities
+ Achieves more accurate results compared to several pairwise methods and can align large point sets without subsampling
+ Boundary conditions can be mapped to masses (*e.g.*, prior matches)

3DV

Thank You

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Thank You