A Quantum Computational Approach to Correspondence Problems on Point Sets

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http://gvv.mpi-inf.mpg.de/projects/QA/

Motivation and Contributions

+ Quantum computers are already used to solve difficult combinatorial optimisation problems, and they can be useful in computer vision
+ We show that the classical problem of *finding optimal transformation and correspondences between two point sets* can be efficiently solved on a quantum computer. The quantum annealing time is constant and does not depend on the size of the inputs in a given dimension.
+ We show how to formulate point set alignment as a *quadratic binary unconstrained optimisation problem* (QUBOP) of the form

$$\arg\min_{\mathbf{q}\in\mathbf{B}^n} \mathbf{q}^\mathsf{T}\mathbf{P}\mathbf{q},$$

and overcome the difficulty of rotation parametrisation.

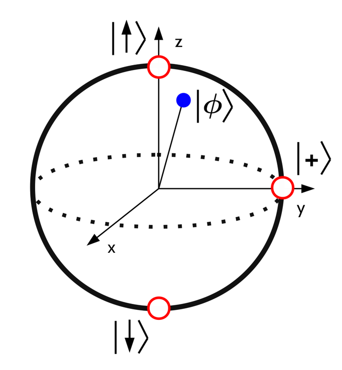
What is a Quantum Computer?

+ Quantum computers take advantage of quantum mechanical effects, *i.e.*, quantum superposition, entanglement and tunnelling [1, 2].
+ They can perform all operations which classical computers can perform, plus multiple algorithms which have lower complexity class compared to their classical counterparts (*e.g.*, prime number factorisation [3])
+ Quantum computers can be classified into two models – gates model and quantum annealers [4]
+ Quantum annealing machines relying on the *adiabatic theorem of quantum mechanics* [5] are called adiabatic quantum computers (AQC)

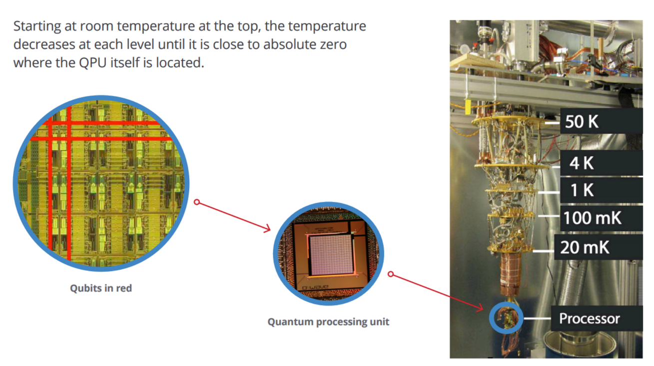
Adiabatic Quantum Annealers

$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$

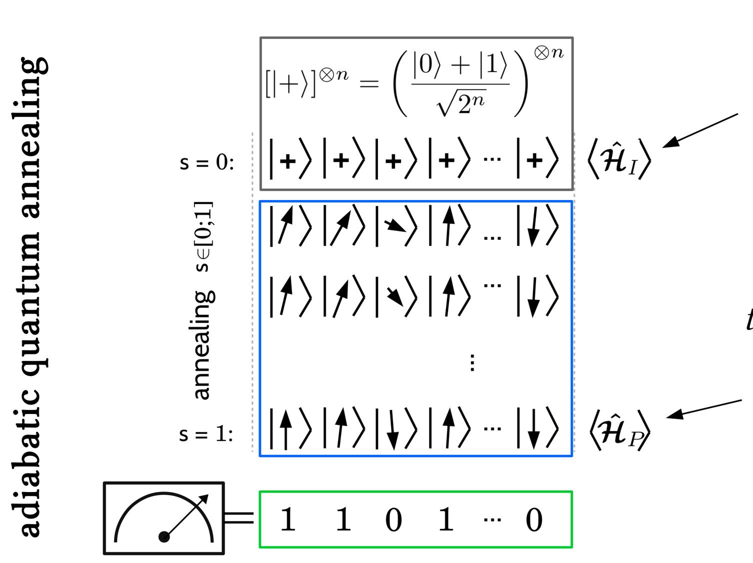
$|\alpha|^2 + |\beta|^2 = 1$



model of a qubit (Bloch sphere)



D-WAVE 2000Q with ~2k qubits [6]



initialisation (initial default Hamiltonian)

annealing (20 μs.)

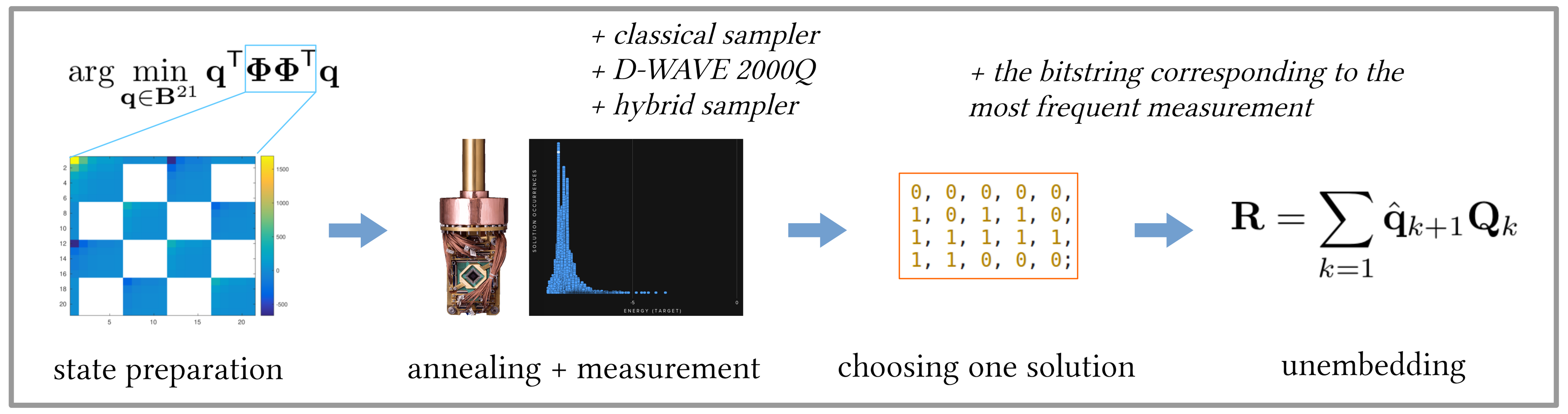
the system evolves to the ground state of the problem Hamiltonian

measurement

classical bitstring (ready for unembedding)

qubits collapse to one of the basis states

Quantum Approach (QA)



State Preparation:: Transformation Estimation

Reference point set: $[\mathbf{x}_n] \in \mathbf{X} \in \mathbb{R}^{D\times N}$

Template point set: $[\mathbf{y}_n] \in \mathbf{Y} \in \mathbb{R}^{D\times N}$

Power series of $\mathbf{R}$, $\mathbf{R}^{-1} = \mathbf{R}^\mathsf{T}$:

$$\mathbf{S} = \theta\,\mathbf{M},\ \mathbf{M} = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \text{(2D)},\quad \begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix} \text{(3D)}$$

Cayley-Hamilton Theorem:

$$\mathbf{S}^2 + \theta^2\mathbf{I} = 0 \quad \text{(2D)} \qquad -\mathbf{S}^3 - \theta^2\mathbf{S} = 0 \quad \text{(3D)}$$

Exponential map for $\mathbf{R}$ with power series:

$$\mathbf{R} = \exp(\mathbf{S}) = \begin{cases} \cos(\theta)\,\mathbf{I} + \left(\frac{\sin(\theta)}{\theta}\right)\mathbf{S} \\ \mathbf{I} + \left(\frac{\sin\theta}{\theta}\right)\mathbf{S} + \left(\frac{1-\cos\theta}{\theta^2}\right)\mathbf{S}^2 \end{cases} = \begin{cases} \cos(\theta)\,\mathbf{I} + \sin(\theta)\,\mathbf{M} & \text{(2D)} \\ \mathbf{I} + \sin\theta\,\mathbf{M} + (1-\cos\theta)\,\mathbf{M}^2 & \text{(3D)} \end{cases}$$

It leads to our basis for $\mathbf{R}$:

$$\{\mathbf{Q}_k = \omega\,\mathbf{C} \in \mathbb{R}^{2\times 2}, \forall\omega \in \{0.5, 0.2, 0.1, 0.1, 0.05\}, \forall\mathbf{C} \in \{\mathbf{I}, \mathbf{M}, -\mathbf{I}, -\mathbf{M}\}\}. \quad \text{(2D)}$$

$$\{\mathbf{Q}_k^{3D} = \omega\,\mathbf{C}^{3D} \in \mathbb{R}^{3\times 3}, \forall\omega \in \{0.5, 0.2, 0.1, 0.1, 0.05\}, \forall\mathbf{C}^{3D} \in \{\mathbf{I}, -\mathbf{I}, \mathbf{M}_a, -\mathbf{M}_a, \mathbf{M}_b, -\mathbf{M}_b, \mathbf{M}_c, -\mathbf{M}_c, \mathbf{M}_d, -\mathbf{M}_d, \mathbf{M}_e, -\mathbf{M}_e, \mathbf{M}_f, -\mathbf{M}_f\}\}. \quad \text{(3D)}$$

$$\mathbf{M} = a\underbrace{\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}}_{\mathbf{M}_a} + b\underbrace{\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ -1 & 0 & 0\end{bmatrix}}_{\mathbf{M}_b} + c\underbrace{\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}}_{\mathbf{M}_c} \quad \text{(basis for M)}$$

$$\mathbf{M}_d = \begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \mathbf{M}_e = \begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}, \mathbf{M}_f = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix} \quad \text{(basis for } \mathbf{M}^2\text{)}$$

The final QUBOP and $\mathbf{P}$:

$$\arg\min_{\mathbf{q}\in\mathbf{B}^{21}} \mathbf{q}^\mathsf{T}\mathbf{\Phi}\mathbf{\Phi}^\mathsf{T}\mathbf{q} \qquad \mathbf{P} = \mathbf{\Phi}\mathbf{\Phi}^\mathsf{T} \quad \text{(2D)}$$

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{x}_1^\mathsf{T} & \mathbf{x}_2^\mathsf{T} & \dots & \mathbf{x}_N^\mathsf{T} \\ -[\mathbf{Q}_1\mathbf{y}_1]^\mathsf{T} & -[\mathbf{Q}_1\mathbf{y}_2]^\mathsf{T} & \dots & -[\mathbf{Q}_1\mathbf{y}_N]^\mathsf{T} \\ -[\mathbf{Q}_2\mathbf{y}_1]^\mathsf{T} & -[\mathbf{Q}_2\mathbf{y}_2]^\mathsf{T} & \dots & -[\mathbf{Q}_2\mathbf{y}_N]^\mathsf{T} \\ \vdots & \vdots & \ddots & \vdots \\ -[\mathbf{Q}_K\mathbf{y}_1]^\mathsf{T} & -[\mathbf{Q}_K\mathbf{y}_2]^\mathsf{T} & \dots & -[\mathbf{Q}_K\mathbf{y}_N]^\mathsf{T} \end{bmatrix}$$

State Preparation:: Point Set Alignment

Minimise *gravitational potential energy* [7] of the system of particles (reference + template) with local point linking:

$$\mathbf{E}(\mathbf{R},\mathbf{t}) = \sum_m\sum_n \mu_{\mathbf{y}_m}\,\mu_{\mathbf{x}_n}\,\|\mathbf{R}\,\mathbf{y}_m + \mathbf{t} - \mathbf{x}_n\|_2$$

$$\arg\min_{\mathbf{q}\in\mathbf{B}^{21}} \mathbf{q}^\mathsf{T}\mathbf{\Phi}\mathbf{\Phi}^\mathsf{T}\mathbf{q} \qquad \mathbf{\Phi} = [\mathbf{\Phi}_1\mathbf{\Phi}_2\dots\mathbf{\Phi}_N] \quad \text{(2D)}$$

For each template point:

$$\mathbf{\Phi}_n = \begin{bmatrix} \mathbf{x}_n^\mathsf{T} & \mathbf{x}_n^\mathsf{T} & \dots & \mathbf{x}_n^\mathsf{T} \\ -[\mathbf{Q}_1\mathbf{y}_1^n]^\mathsf{T} & -[\mathbf{Q}_1\mathbf{y}_2^n]^\mathsf{T} & \dots & -[\mathbf{Q}_1\mathbf{y}_{L(n)}^n]^\mathsf{T} \\ -[\mathbf{Q}_2\mathbf{y}_1^n]^\mathsf{T} & -[\mathbf{Q}_2\mathbf{y}_2^n]^\mathsf{T} & \dots & -[\mathbf{Q}_2\mathbf{y}_{L(n)}^n]^\mathsf{T} \\ \vdots & \vdots & \ddots & \vdots \\ -[\mathbf{Q}_K\mathbf{y}_1^n]^\mathsf{T} & -[\mathbf{Q}_K\mathbf{y}_2^n]^\mathsf{T} & \dots & -[\mathbf{Q}_K\mathbf{y}_{L(n)}^n]^\mathsf{T} \end{bmatrix}$$

Unembedding

Unembedding is the decoding of the solution to QUBOP to the solution of the original alignment problem:

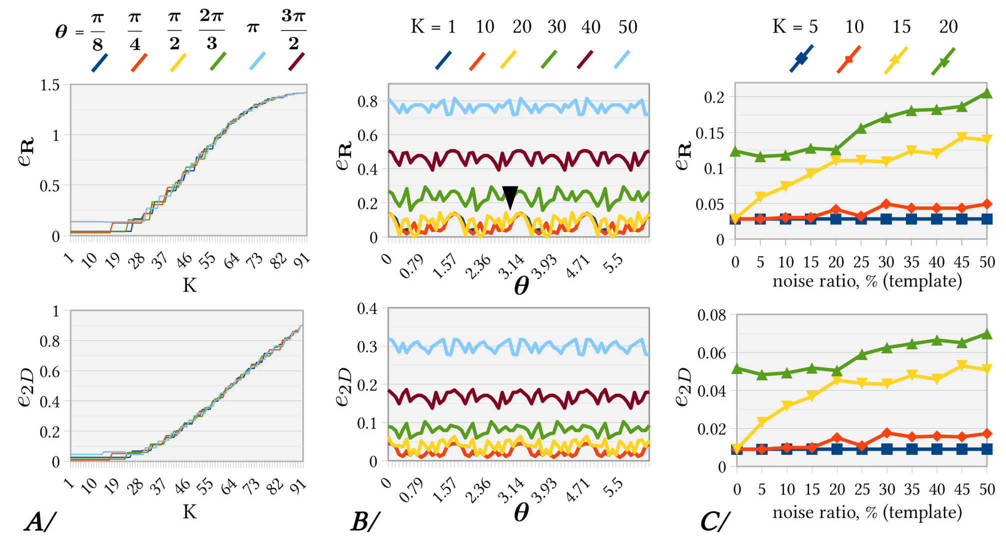
$$\mathbf{R} = \sum_{k=1}^{K} \hat{\mathbf{q}}_{k+1}\mathbf{Q}_k,$$

where a classical bitstring $\hat{\mathbf{q}}$ is the measurement of $\mathbf{q}$.

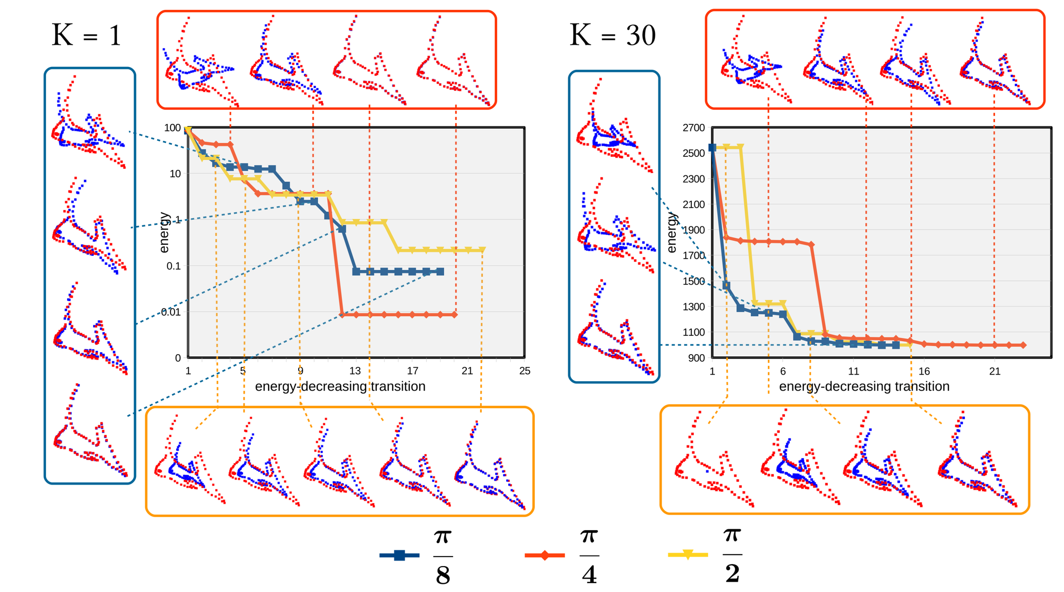
If required, the obtained approximate solution can be projected to the rotation group:

$$\min \|\mathbf{R}_r - \mathbf{R}\|^2_{\mathcal{HS}}, \quad \text{s. t. } \mathbf{R}_r^{-1} = \mathbf{R}_r^\mathsf{T} \text{ and } \det(\mathbf{R}_r) = 1.$$

Tests on CPU Sampler and Spectral Gap Analysis (2D)

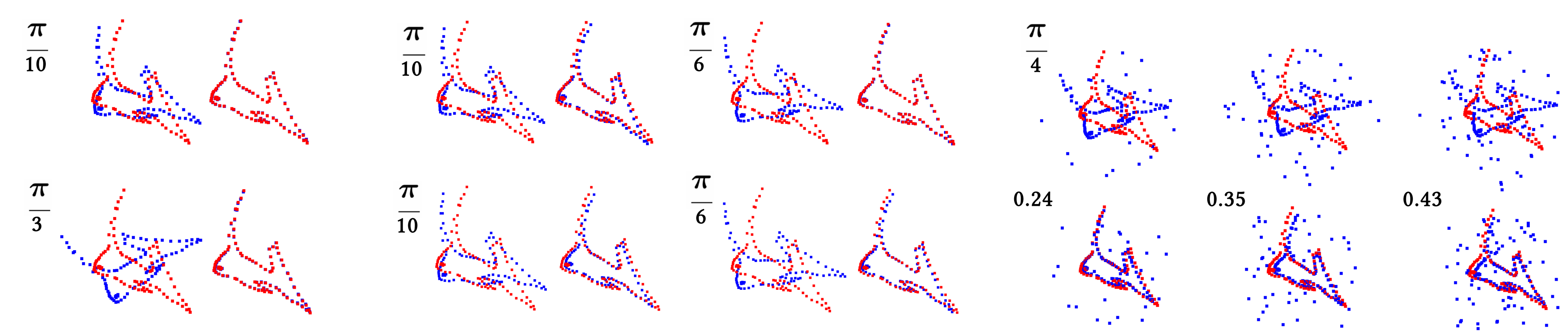


The metrics as the functions of A/: the size of the point interaction region parametrised by K; B/: the angle of initial misalignment θ; C/: the template noise ratio.



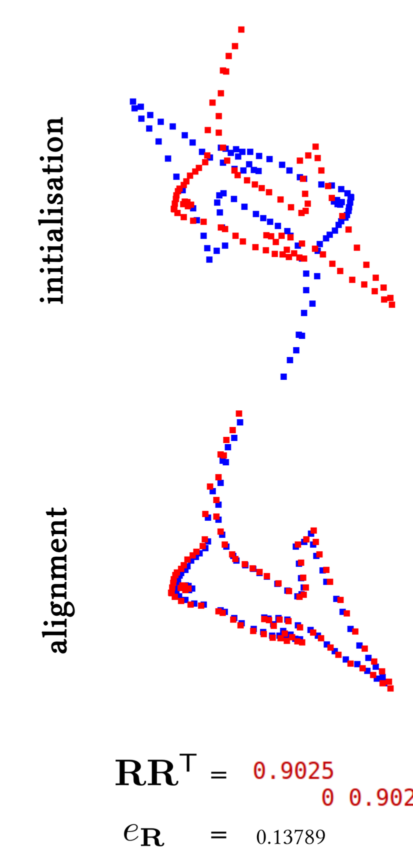
The sequences of energy-decreasing transitions and the corresponding energy values observed in our sampler.

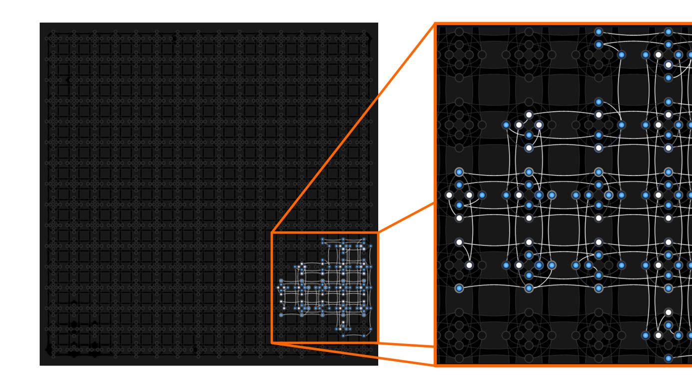
Experiments on D-WAVE 2000Q (2D)



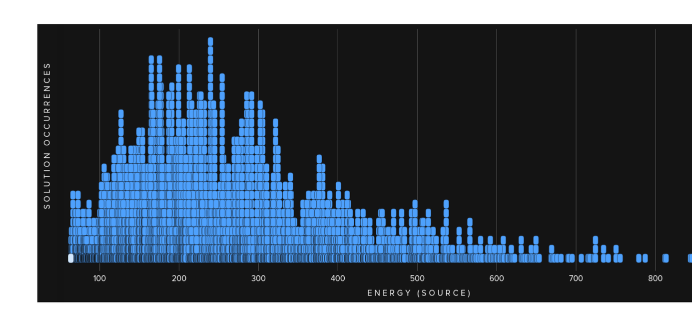
K = 1   K = 20 (top row), 30 (bottom row)   K = 30, template noise

Minor-Embedding and Quantum Annealing

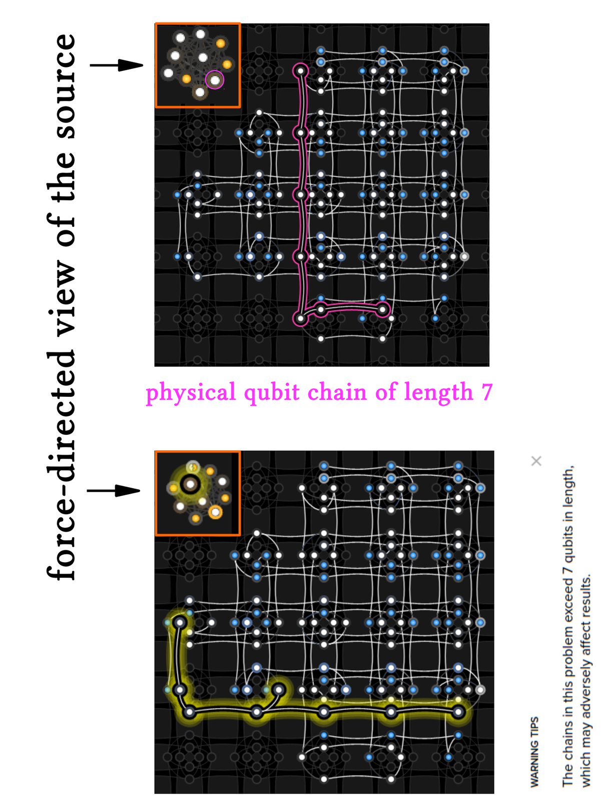




entire *Chimera* graph with [16 x 16] unit cells   the region with the embedded problem



distribution of solutions after 5k runs



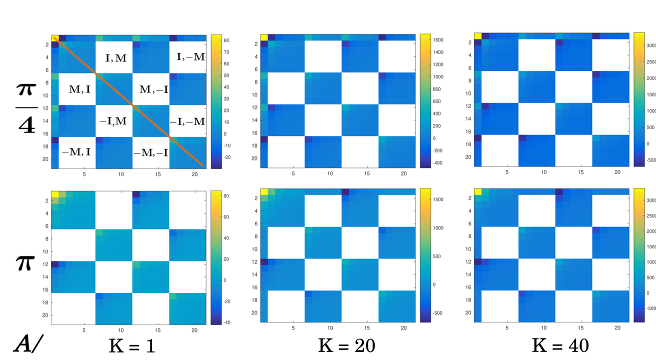
mapping of the problem

quantum notion	classical counterpart
qubit (states $\|0\rangle$ and $\|1\rangle$)	bit (states 0 and 1)
(time-dependent) Hamiltonian	energy functional
eigenstate	some energy state
ground state	globally optimal energy state
quantum system evolution	optimisation process
quantum annealing	simulated annealing

Quantum notions and their classical counterparts.

	TE	*K*				
		10	20	30	40	50
e_{2D}	0.023	0.026	0.041	0.078	0.17	0.3
σ_{2D}	0.012	0.013	0.012	0.012	0.012	0.013
$e_\mathbf{R}$	0.058	0.062	0.083	0.22	0.47	0.764
$\sigma_\mathbf{R}$	0.041	0.044	0.041	0.036	0.031	0.03

The accuracy of QA under random initial misalignments.



Examples of $\mathbf{P} = \mathbf{\Phi}\mathbf{\Phi}^\mathsf{T}$ in QA.

Complexity of state preparation:

$\mathcal{O}(K^2DN)$ (transformation estimation)

$\mathcal{O}(K^2DN\bar{L})$ (point set alignment)

Error metrics:

$$e_{2D} = \frac{\|\mathbf{RY}-\mathbf{X}\|_{\mathcal{HS}}}{\|\mathbf{X}\|_{\mathcal{HS}}} \quad \text{(alignment error)}$$

$$e_\mathbf{R} = \left\|\mathbf{I} - \mathbf{RR}^\mathsf{T}\right\|_{\mathcal{HS}} \quad \text{(transformation discrepancy)}$$

References

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